Lebanese University
Faculty of information
Section 1 **Partial Exam: Discrete Mathematics** **Time: 60 minutes**

Problem 1.

1. Show that these conditional statements are logically equivalent.
   a) $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$
   b) $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$

2. Show that each of these conditional statements is a tautology by using truth tables.
   a) $(p \wedge q) \to (p \vee q)$
   b) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

3. Determine whether each of these compound propositions is satisfiable.
   a) $(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge \neg p \to \neg q)$

4. Let $Q(x)$ be the statement "$x + 1 >$ [illegible]" If the domain consists of all integers, what are these truth values?

   a) $Q(0)$ b) $Q(-1)$ c) $Q(1)$
   d) $\exists x\, Q(x)$. e) $\forall x\, Q(x)$ f) $\exists x \neg Q(x)$
   g) $\forall x \neg Q(x)$

Problem 2.
Part A.

1. Suppose A, B and C are sets. Show that:
   a) $A - (A \cap B) = A - B$
   b) $A \times (B - C) = (A \times B) - (A \times C)$

2. Let $f: A \to B$ and $g: B \to C$ two applications.
   a) Show that if gof injective and f surjective then g is injective.
   b) Show that if f surjective and g surjective then gof is surjective.

3. Determine which of the following functions are one-to-one and which are onto. Justify your answers with arguments and counter arguments.
   a) $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 - x$
   b) $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |1 - 3x|$

4. Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

Find

a) $A \vee B$. b) $A \wedge B$. c) $A \odot B$.

Good Luck